Dynamic Characterization of Noise and Vibration Transmission Paths in Linear Cyclic Systems (I)

— Theory —

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Linear cyclic systems (LCS's) are a class of systems whose dynamic behavior changes cyclically. Such cyclic behavior is ubiquitous in systems with fundamentally repetitive motions (e. g. all rotating machinery). Yet, the knowledge of the noise and vibration transmission paths in LCS's is quite limited due to the time-varying nature of their dynamics. The first part of this two-part paper derives a generic expression that describes how the noise and/or vibration are transmitted between two (or multiple) locations in the LCS's. An analysis via the Fourier series and Fourier transform (FT) plays a major role in deriving this expression that turns out to be transfer function dependent upon the cycle position of the system. The cyclic nature of the LCS' transfer functions is shown to generate a series of amplitude modulated input signals whose carrier frequencies are harmonic multiples of the LCS' fundamental frequency. Applicability of signal processing techniques used in the linear time-invariant systems (LTIS's to the general LCSs is also discussed. Then, a criterion is proposed to determine how well a LCS can be approximated as a LTIS. In Part II, experimental validation of the analyses carried out in Part I is provided.

Key Words : Linear Cyclic Systems, Transfer Function, Amplitude Modulation

1. Introduction

Dynamic systems with fundamentally repetitive motion frequently exhibit cyclic behavior and consequently generate periodic mechanical vibration and acoustic radiation, which in turn are transmitted via periodically time-varying media to the surface and air. In most cases, the noise and vibration emanating from these systems turn out to be a nuisance, which need to be either eliminated or at least reduced before they are put into practice. Considering the importance of controlling the noise and vibration in cyclic systems (CS), it is not surprising that numerous papers have addressed the various issues related to this subject such as the noise and vibration signal analysis (Tamaki et al., 1995 ; Barker and Hinich, 1994 ; Harrap and Wang, 1994 ; Cann, 1992 ; Meng and Qu, 1991 ; Young, 1994), the noise and vibration control (Mohamed and Busch-Vishniac, 1995 ; Ha, 1995 ; Oh, 1993), transmission path identification (Lou et al., 1993 ; Rantala and Suoranta, 1991 ; Young, 1994), etc., to cite more recent papers. These works can be categorized into two groups by their assumptions on the transmission path dynamics:

(1) The transmission path dynamics is linear and time-invariant (LTI). The transmission path identification and noise/vibration control works fall into this category. Here, it is assumed a priori

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that the input signal is transmitted through the linear time-invariant systems (LTIS's) to the output without explicit validation.

(2) The transmission path dynamics is irrelevant to the problem under consideration. The noise/vibration analysis works belong here. Since the goal of the works in this category is to analyze the noise and vibration signals for the signal characterization or diagnosis, the transmission path dynamics is of less interest.

The above assessment naturally motivates us to analyze the transmission paths of noise and vibration in CS's without assuming a priori that the transmission paths are the LTIS's. The resulting transmission path analysis may play a role in validating the assumption that the transmission paths are the LTIS and understanding how the input signals are related to the output signals, which obviously helps to understand the input/output signals themselves. Multiplicative behavior of the generated mechanical vibro/acoustic radiation and the transmission paths (both cyclically time-varying) does not seem to allow such general cyclic system (CS) to be readily analyzable, which partially explains lack of understanding of the signals and transmission paths in the general CS's. On the other hand, linear cyclic systems (LCS's) are a subclass of the general CS's, whose transmission media are linear and cyclic. Their relatively simple and analytically tractable dynamics makes the analysis of LCS' signals and transmission paths more amenable. Though not exhaustive, LCS's account for most of the CS's as far as transmission paths are concerned. Once the scope of this paper is confined to the LCS's, the linearity of the transmission paths makes it possible to represent the output signals as the convolution integrals (in time-domain) of the input signals and (cyclically) time-varying impulse response functions (Bracewell, 1986). Then, via the Fourier transform and Fourier series analyses, the convolution integrals are further simplified as input-output transfer functions in the frequency domain, from which various subsequent analyses can be carried out. Though well-known to rotating machinery researchers (Baker and Hinich, 1994 ; Meng and Qu, 1991), it can be shown that the LCS's generate a series of amplitude modulated input signals, where the carrier frequencies are the harmonic multiples of the fundamental frequency of the LCS's. The LCS's can be classified into two groups based on the relative magnitudes of the carriers: genuine linear cyclic systems (GLCS's) and pseudo linear cyclic systems (PLCS's). A subsequent analysis leads to a criterion to determine how well a given LCS can be approximated as an LTIS. The PLCS's can be well approximated as the LTIS's, while the GLCS's in general cannot.

Section 2 derives the expression describing the input/output relation of an LCS, where the Fourier transform and Fourier series analyses provide two indispensable tools. Section 3 explains the behaviors of the GLCS's and the applicability of the signal processing techniques for the LTIS's to the GLCS's. Section 4 shows the analysis for the PLCS's, similar to that in Sec. 3. A criterion to determine how well a given LCS can be approximated as an LTIS is also proposed in Sec. 5.

2. Input/Output Relation of Linear Cyclic Systems

In this section, a mathematical description of the noise and vibration transmission path in a simple LCS is derived. Figure 1 shows the sche matic of an LCS that consists of three components : an inner sphere, rotating ellipse, and outer spherical shell. The gap between the inner sphere and outer shell is usually filled with any linear media such as refrigerant, gasoline, air, etc. The system is simple but captures the essence of the LCS's in terms of the noise and vibration transmission, which is required for the analysis in this paper to be valid for the for, more complex LCS's. Assume that the system (or to be specific, the ellipse) is running at an angular frequency ω. Define an abstract entity (or angle) θ as an indicator of the system status during the repetitive (or cyclic) motion. As the system goes through a full cycle, θ increases monotonically from 0°to 360°.In real-world rotating machines such as an automobile

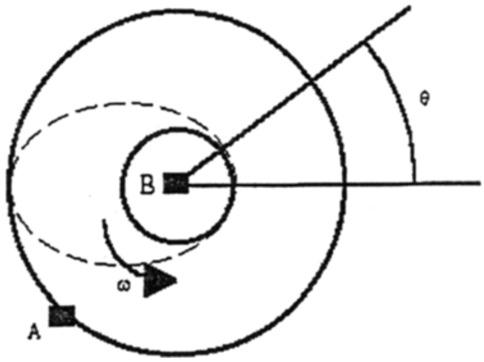

Fig. 1 The schematic of a linear cyclic system.

engine, the crank angle can be chosen as θ. The transducers 'A' and 'B' in Fig. 1 are used either to instrument the LCS or to excite the LCS. The transducers 'A' and 'B' are located externally on the shell and internally on the inner sphere, respectively, which are the fixed points of the LCS (it would be difficult to place a sensor(s) or an actuator(s) on moving parts of the system). Specific requirements for analysis determine the types of transducers. Note that the present analysis remains valid for different types of transducers, even though only two types of transducers, a force transducer and an accelerometer, are used here. The external transducer is used as the input transducer throughout the analysis in this section for simplicity, even if the same analysis holds true when the internal transducer is used as the input transducer. Throughout this paper, the distinction between the forward path (from 'B' to 'A') and reverse path (from 'A' to 'B') in Fig. 1 is not made explicitly unless specified otherwise, since the most analyses are independent of the path direction. In most practical mechanical systems, the noise/vibration is transmitted along the forward path. The periodic noise/vibration signals go through the LCS, resulting in a multiplicative interaction between the source signals and the transmission media. The reverse path usually becomes of interest when signal processing techniques such as the transfer function estimation via the reciprocity principle (Kinsler et al., 1982 ; Belousov and Rimskii-Korsakov 1975 ; Ten Wolde, 1973 ; Ten Wolde, 1976) are applied to identify the noise/vibration transmission paths instead of the direct identification of the forward path, while there may well exist some mechanical systems where the noise/vibration is transmitted along the reverse path. When applying the reciprocity principle, it is usual to apply the force on the shell (at the transducer 'A' in Fig. 1) and measure the response inside the LCS (at the transducer 'B' in Fig. 1), mainly for the sake of readily deployable experiment.

We start the analysis of the LCS by defining its impulse response. Assume that the LCS in Fig. 1 has reached the steady state[1]. While the LCS is running, an impulse is applied at the external transducer 'A' (a force transducer) at an angle θ_0. In the meantime, the time history of the acceleration measurement at the internal transducer 'B' (an accelerometer) is recorded. Then, the recorded time history is defined as the impulse response of the LCS, $h(\theta_0, t)$, at θ_0. It is worthwhile to note that the LCS is still running and changing its angular position θ during the acceleration measurement. θ_0 is the system status angle when the impulse is applied at the force transducer 'A'. The second argument t is the elapsed time from the instant when the impulse is applied. Our definition of the impulse response of a linear time-varying system differs from others (see Bracewell, 1986 ; Kailath, 1980 and references therein) in that the second argument t is the elapsed time from the moment when the impulse is applied rather than the time elapsed after the system starts to run. It is shown later that this way of defining the impulse response significantly simplifies the subsequent analysis.

Once the impulse response is defined, it is straightforward to express the input/output relation for the LCS. Denote the input (transducer 'A' reading) and output (transducer 'B' reading) at time t as $u(t)$ and $y(t)$, respectively. Then the output $y(t)$ becomes (Bracewell, 1986 ; Kailath, 1980)

[1] This implies that the system has been running for much longer period of time than its slowest time constant. Since the noise/vibration transmission path analysis is of utmost interest to us, the slowest time constant should be interpreted in the context of the noise/vibration transmission.

$$y(t)=\int_{-\infty}^{t}h[\theta(\tau),\ t-\tau]u(\tau)\,d\tau, \qquad (1)$$

where $\theta(t)=\theta_0+2\pi\int_0^t f_r(t')\,dt'$ and $f_r(t)$ is the instantaneous frequency of the LCS at time t. Note that the variation of the instantaneous frequency (or rotation speed) often occurs within a cycle of the LCS. The argument t is dropped for simplicity in the following analysis unless needed for clarification. $\theta(t)$ is periodic with a fundamental period T_R. In other words, $\theta(t)$ crosses the same angular position (with modulo 2π) at every T_l seconds. Also define $f_R(=1/T_R)$ as the average (or equivalent) fundamental frequency.

Taking the Fourier Transform (FT) of Eq. (1) yields

$$\begin{aligned}Y(f) &\overset{\text{def}}{=} F[y(t)]\\ &=\int_{-\infty}^{\infty}\int_{-\infty}^{t}h[\theta(\tau),\ t-\tau]\\ &\quad u(\tau)\,d\tau\exp(-j2\pi ft)\,dt \qquad (2)\end{aligned}$$

Then changing the order of double int egral results in

$$\begin{aligned}Y(f) &=\int_{-\infty}^{\infty}\int_{\tau}^{\infty}h[\theta(\tau),\ t-\tau]u(\tau)\exp\\ &\quad(-j2\pi ft)\,dtd\tau \qquad (3)\\ &=\int_{-\infty}^{\infty}\int_{\tau}^{\infty}h[\theta(\tau),\ t-\tau]\exp[-j2\pi f\\ &\quad(t-\tau)]dtu(\tau)\exp(-j2\pi f\tau)\,d\tau \qquad (4)\end{aligned}$$

Introducing a new variable $p=t-r$ gives

$$\begin{aligned}Y(f) &=\int_{-\infty}^{\infty}\int_{0}^{\infty}h[\theta(\tau),\ p]\exp(-j2\pi fp)\\ &\quad dpu(\tau)\exp(-j2\pi f\tau)\,d\tau \qquad (5)\\ &=\int_{-\infty}^{\infty}H[\theta(\tau),f]u(\tau)\exp(-j2\pi f\tau)\,d\tau, \qquad (6)\end{aligned}$$

$$\begin{aligned}H[\theta(\tau),f] &\overset{\text{def}}{=}\int_{0}^{\infty}h[\theta(\tau),\ p]\\ &\quad\exp(-j2\pi fp)\,dp\\ &=\int_{-\infty}^{\infty}h[\theta(\tau),\ p]\exp(-j2\pi fp)\,dp,\end{aligned}$$

where the last equality comes from the causality of $h(\theta,\ t)$. In other words, $h(\theta,\ t)=0$ for $t<0$, i. e., the LCS does not respond before it is excited. Note that $H(\theta,\ f)$ is the FT of $h(\theta,\ t)$ with respect to its second argument t and is the FT of the impulse response, where the impulse is applied at angle θ. Then, the periodicity of $H(\theta,\ f)$ with respect to its first argument θ (with the fundamental period T_R) allows the following Fourier expansion (with respect to θ):

$$\begin{aligned}H(\theta,\ f) &=H[\theta(\tau),\ f]\\ &=\sum_{k=-\infty}^{\infty}A_k(f)\exp(j2\pi kf_R\tau) \qquad (7)\end{aligned}$$

$$\begin{aligned}A_k(f) &=\int_{-\phi/T_R}^{\pi/T_R}H[\theta(q),\ f]\exp\\ &\quad(-j2\pi kf_Rq)\,d_R \qquad (8)\end{aligned}$$

where $A_k(f)$ is the Fourier coefficient corresponding to the k^{th} harmonic at the frequency f.

Equations (7) and (8) turn out to be es sential to the analyses of two representative classes of the LCS's described in Sec. 3 and 4. The characteristics of the LCS's depend heavily upon the magnitude of $A_k(f)$. For example, if $A_k(f)$ is identically zero for all $k(\neq 0)$ at any given frequency f, the corresponding LCS become LTIS. On the other hand, if $A_k(f)$ is significant for some $k(\neq 0)$, then the behavior of the LCS becomes complicated and needs careful treatment. Two extreme but still representative cases are considered in this paper:

1. $A_k(f)$ is significant for some $k(\neq 0)$ at a given frequency f.
2. $A_k(f)$ is relatively small for all $k(\neq 0)$ at a given frequency f.

The corresponding systems may be classified as a genuine LCS and a pseudo LCS, respectively. Above rather ad-hoc criterion i s more rigorously presented in Sec. 5.

3. Genuine Linear Cyclic Systems

We first examine the transfer function $H(\theta,f)$ itself for a given f. The genuine linear cyclic systems (GLCS's) have been defined such that there exists some significant $A_k(f)$'s for a given frequency f, which implies that there exists significant periodic fluctuations of the transfer function $H(\theta,f)$. A number of significant $A_k(f)$'s $(k\neq 0)$ and k together determine how the transfer function $H(\theta,f)$ fluctuates over a cycle at frequency f. A smaller number of significant $A_k(f)$'s means that for a given frequency$f$, $H(\theta,\ f)$ changes its value like a sinusoid with respect to θ. A larger number of significant $A_k(f)$'s indicates that for a given f, $H(\theta, f)$ goes through an abrupt fluctuation(s) with respect to θ. For a given frequency f, smaller k's tend to cause $H(\theta, f)$ to fluctuate very slowly over a cycle, while larger k's force $H(\theta, f)$ to evolve very quickly over a cycle. The interaction among $A_k(f)$'s for different f's is discussed later in this section.

Now the effect of the cyclically time-varying transfer function $H(\theta, f)$ of the GLCS's on the input/output relation is evaluated. Substituting Eq. (7) into Eq. (6) yields

$$Y(f)=\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}A_k(f)\exp(j2\pi kf_R\tau)\,u(\tau)\exp(-j2\pi f\tau)\,d\tau \quad (9)$$

$$=\sum_{k=-\infty}^{\infty}A_k(f)\int_{-\infty}^{\infty}u(\tau)\exp[-j2\pi(f-kf_R)\tau]d\tau \quad (10)$$

For simplicity, only three terms corresponding to $k=-1$, 0, 1 are assumed to be significant. The analysis can be readily extended to a general case in which more than three terms are significant. Then Eq. (10) becomes

$$Y(f)=\sum_{k=-1}^{1}A_k(f)\int_{-\infty}^{\infty}u(\tau)\exp[-j2\pi(f-kf_R)\tau]d\tau \quad (11)$$

Eq. (11) can be readily interpreted as the $A_k(f)$-weighted sum of the FT of three amplitude modulated (AM) signals with the input s ignal $u(\tau)$ and the carrier frequencies $-f_R$, 0, f_R. By modulation theorem of the FT, the spectrum of $Y(f)$ has three $A_k(f)$-weighted replicas of the baseband spectrum of $U(f)\{=F[u(\cdot)]\}$, shifted by $-f_R$, 0, f_R (Bracewell, 1986). This result obviously applies to a GLCS excited by a single sinusoid input with the unit amplitude at the frequency f_e. Since the modulation property is independent of the excitation input, the input sinusoid at f_e is modulated with the carriers at the frequencies $-f_R$, 0, f_R. As a result, three sinusoids with amplitudes $A_{-1}(f_e-f_R)$, $A_0(f_e)$, $A_1(f_e+f_R)$ at the frequencies f_e-f_R, f_e, f_e+f_R are observed at the output. Generating the three frequency shifted replicas of the input spectrum is not possible for the LTIS's. The LTIS's cannot generate any frequency component that does not exist at the input.

Up until now, a clear distinction has not been made between the two transmission paths: (1) the forward path along which the LCS-generated noise (at the transducer 'B') is transmitted to the shell (at the transducer 'A') , and (2) the reverse path along which an externally excited signal (' A') is transmitted to the internal transducer ('B'). The reason is that the same argument holds true for both paths. Now it remains to examine distinctive features of each path and to show whether the two paths are equivalent, i. e., the transfer functions[2] along the two paths are identical. The equivalence of the two paths is particularly important when the forward path cannot be readily identified, while the reverse path can be identified, e. g. through the reciprocity principle. Remember that the system under consideration has to be the LTI and passive in order for the reciprocity principle to be applicable (Kinsler et al., 1982 ; Belousov and Rimskii-Korsakov 1975 ; Ten Wolde, 1973 ; Ten Wolde, 1976).

The reverse path is first analyzed here due to its simplicity. In the reverse path, only one excitation frequency exists. Three amplitude modulated sinusoids would be observed at the output. These sinusoids are now examined in some detail. Exciting a GLCS with a sinusoid at the frequency f_e would give

$$Y(f)=\sum_{k=-1}^{1}A_k(f)\int_{-\infty}^{\infty}\exp[j2\pi f_e(\tau+t_0)]\exp(j2\pi kf_R\tau)\exp(-j2\pi f\tau)\,d\tau \quad (12)$$

where t_0 is the time lag of the input sinusoid relative to the reference quantity of the system cyclic behavior (e. g. the crank angle in rotating machines). With the input sinusoid as a reference, Eq. (12) becomes

$$Y(f)=\sum_{k=-1}^{1}A_k(f,\ t_0)\int_{-\infty}^{\infty}\exp(j2\pi f_e\tau)\exp(j2\pi kf_R\tau)\exp(-j2\pi f\tau)\,d\tau$$

[2] To be rigorous, it should be regarded merely as the frequency response function since the transfer function cannot be defined for the GLCS's in that multiple signals at different frequencies due to amplitude modulation appears at the output when the system is excited with a sinusoid containing a single frequency. The concept of the transfer function applies to only the LTIS's.

$$= \sum_{k=-1}^{1} A_k(f, t_0) \int_{-\infty}^{\infty} \exp[j2\pi(f_e + kf_R - f)\tau] d\tau \quad (13)$$

where

$$\begin{aligned} A_k(f, t_0) &= \int_{-\pi/T_R}^{\pi/T_R} H[\theta(q+t_0), f] \exp(-j2\pi k f_R q)\, dq, \\ &= \int_{-\pi/T_R+t_0}^{\pi/T_R+t_0} H[\theta(q), f] \exp(-j2\pi k f_R q) \exp(j2\pi k f_R t_0)\, dq, \\ &= \int_{-\pi/T_R}^{\pi/T_R} H[\theta(q), f] \exp(-j2\pi k f_R q) \exp(j2\pi k f_R t_0)\, dq, \\ &= \int_{-\pi/T_R}^{\pi/T_R} H[\theta(q), f] \exp(-j2\pi k f_R q)\, dq \exp(j2\pi k f_R t_0) \quad (14) \end{aligned}$$

where the periodicity of $H(\theta, f)\exp(-j2\pi k f_R q)$ with respect to q was used in the derivation. Note that $A_k(f)$ depends on the time lag t_0 for $k \neq 0$. However, for $k=0$, $A_k(f, t_0)$ does not depend on t_0. For $k \neq 0$, the magnitudes of $A_k(f, t_0)$ do not change but the phases change as $\exp(j2\pi k f_R t_0)$. This can be summarized as follows: When a sinusoid is applied at the frequency f_e at several randomly-selected time instants t_0, time invariant $A_0(f_e, t_0)$ is observed at f_e but time-varying $A_k(f_e + kf_R, t_0)$ at $f_e + k_{fR}$, for $k \neq 0$. This is experimentally demonstrated later in Part II of this two-part paper as a partial validation of our analysis.

Then, the forward path is analyzed. A GLCS generates the internal noise with multiple harmonics of the revolution frequency f_R through various mechanisms, which is in turn transmitted to the external transducer through the GLCS, which modulates the generated internal noise, where the carrier frequencies are the harmonic multiples of f_R. This implies that multiple sinusoids at different frequencies may contribute to the output for a given frequency f, which leads us to the conclusion that the GLCS is not an LTIS. However, it is worth noting that the evolution of the excitation signal (internal noise) is synchronized with that of the system characteristics in the forward path.

One may be tempted here to argue that the GLCS in the forward path is an LTIS in that the input/output spectra are well-defined (i. e. they are time-invariant) and their relation can be described by the transfer function of an LTIS, which, of course, is proven false.

From the discussion above, it goes without saying that the two paths are fundamentally different. The above result implies that signal processing techniques based on the assumption that the GLCS's are the LTIS's would not give any meaningful answer.

4. Pseudo Linear Cyclic Systems

The pseudo linear cyclic systems (PLCS's) have been defined such that $A_k(f)$ is relatively small for all k at a given frequency f. This means that the variation of the transfer function over a cycle at the frequency f is minimal. In the time domain, this implies that the impulse response does not appreciably change as the LCS goes through its cycle.

The mathematical analysis starts with Eq. (6) and Eq. (7). Since $A_k(f)$ is relatively small, Eq. (7) can be approximated as

$$\begin{aligned} H(\theta, f) = H[\theta(\tau), f] &\approx A_0(f) \\ &= \int_{-\pi/T_R}^{\pi/T_R} H[\theta(\tau), f] d\tau \end{aligned} \quad (15)$$

where only the mean value of $H[\theta(\tau), f]$ over a cycle is taken to approximate $H[\theta(\tau), f]$. Then, it becomes

$$\begin{aligned} Y(f) &\approx \int_{-\infty}^{\infty} A_0(f)\, u(\tau) \exp(-j2\pi f\tau)\, d\tau \\ &= A_0(f)\, U(f) \end{aligned} \quad (16)$$

where $U(f) = \int_{-\infty}^{\infty} u(\tau) \exp(-j2\pi f\tau)\, d\tau$. This is nothing but the input-output expression of a linear time invariant system (LTIS) in the frequency domain. There is no amplitude modulation as in the case of the GLCS's. A PLCS is essentially an LTIS.

Once the behavior of a PLCS is identified as that of an LTIS, the behavior of the PLCS at a given frequency f_e can be readily predicted. When excited by a single sinusoid at f_e, the PLCS (with negligible sideband signals, to be precise) must have only one sinusoid as a response at the excitation frequency f_e. The transfer function at f_e

is the mean value of $H[\theta(\tau), f_e]$, which does not vary much as θ is varied? Since f_e was arbitrary, this implies that the error would not be significant even if $H[\theta(\tau), f_e]$ for any τ is approximated by $H[\theta(\tau_S), f]$ at a specific time instant τ_S for all f. In the time domain, this subsequently means that the impulse response of the PLCS starting at any specific time instant τ_S would not be different from those starting at any other time instants, which is more rigorously expressed in Section 5. The simple dynamic characteristic of the PLCS makes it dispensable to analyze the forward and reverse paths separately. Then, it remains to show that the forward and reverse paths have the identical transfer functions or the impulse responses. The independence of the transfer functions (or the impulse responses) with respect to cyclic positions effectively eliminates the cyclic behavior of the PLCS, which makes the PLCS's the LTIS's. Most mechanical systems are passive in that they do not increase the noise/vibration energy while transmitting the noise/vibration from one point to another; most mechanical systems dissipate or at most maintain the vibration energy. In this paper, it is assumed without proof that the systems under consideration belong to linear, passive systems. It is well-known that the reciprocity principle holds for a passive LTIS (Kinsler et al., 1982 ; Belousov and Rimskii -Korsakov 1975 ; Ten Wolde, 1973 ; Ten Wolde, 1976). In this respect, the transfer functions for the forward and reverse paths in the PLCS's are identical, which has the following important implication: The sideband criterion presented in Section 5 to classify an LCS either as a GLCS or as a PLCS can be applied to the reverse path instead of the forward path. As noted earlier, experiments along the reverse path make the instrumentation readily deployable during the noise/vibration source and path identification.

Finally, we discuss the applicability of the signal processing techniques to the PLCS's. Since the PLCS's are essentially the LTIS's, any signal processing technique applicable to the LTIS's (e. g. autoregressive modeling in (Rantala and Suoranta, 1991)) can be applied to the PLCS's. In addition, since most mechanical systems (which is of our primary interest in this paper) are passive, signal processing techniques for the passive LTIS's should be applicable here.

5. The Sideband Criterion for LCS

This section presents a criterion to determine how well an LCS can be approximated as an LTIS. Obviously, if an LCS is not an LTIS, it is a GLCS.

5.1 The sideband criterion for LCS

Given an LCS, its impulse response $h(\theta, t)$ and a pre-determined critical number α, the LCS can readily be approximated as an LTIS if the following condition holds:

$$\sup_{[\theta\in 10°,\ 360°]} \int_{-\infty}^{\infty} |h(\theta, t) - \mathrm{avg}_\theta h(\theta, t)|^2 dt < \alpha\ \mathrm{avg}_\theta \int_{-\infty}^{\infty} |h(\theta, t)|^2 dt \tag{17}$$

If the Fourier transform of $h(\theta, t)$, $H(\theta, f)$, is given instead, the above relation Eq. (17) can be equivalently expressed in the frequency domain (using Parseval's theorem (Bracewell, 1986)) as:

$$\sup_{[\theta\in 10°,\ 360°]} \int_{-\infty}^{\infty} |H(\theta, f) - \mathrm{avg}_\theta H(\theta, f)|^2 df < \alpha\ \mathrm{avg}_\theta \int_{-\infty}^{\infty} |H(\theta, f)|^2 df \tag{18}$$

$\sup_x \Psi(x)$ denotes the supremum or maximum of the function Ψ over its argument(s) x. $\Phi(x)$ is the mean value of the function Φ over its argument x. The critical number α is determined a priori. A guideline on choosing α is provided later in this section. Both Eq. (17) and Eq. (18) state that the LCS is a PLCS if the differential energy of the impulse response (or its Fourier transform in the frequency domain) varies by less than $\alpha\cdot$ (its average energy) with respect to θ. In other words, the LCS is an PLCS if the impulse response or its Fourier transform does not change much with respect to θ.

Although compact, the criteria given by Eq. (17) and Eq. (18) are not easy to evaluate. Both the impulse response in the time domain and its Fourier transform in the frequency domain are not easily measurable due to (1) a low signal level compared to the noise/disturbance such as compressor generated noise/vibration, and (2) exhaustive effort required to examine various θ to compute the sup and avg. The low signal level might be avoided by repeating the experiment several times to take an average impulse response $h(\theta, t)$ (with respect to t) and thereby improve the signal-to-noise ratio but, in this case, the repeatability problem raises different issues. The θ variation cannot be resolved without exhaustive experimentation. Eq. (7) allows us to derive a simpler criterion than Eq. (18) (refer to Section 3 to see how readily $A_k(f)$ can be obtained). First, the left-hand side of Eq. (18) is bounded from above as follows:

$$\sup_{\theta}\int_{-\infty}^{\infty}\left|H(\theta(\tau), f)-\text{avg}_{\theta}\, H(\theta, f)\right|^2 df$$
$$=\sup_{\tau}\int_{-\infty}^{\infty}\left|\sum_{k=-\infty}^{\infty} A_k(f)\exp(j2\pi k f_R \tau)-A_0(f)\right|^2 df$$
$$\leq\sup_{\tau}\int_{-\infty}^{\infty}\left[\sum_{k\neq 0}|A_k(f)\exp(j2\pi k f_R \tau)|\right]^2 df$$
$$\leq\int_{-\infty}^{\infty}\left[\sum_{k\neq 0}|A_k(f)|\right]^2 df$$

where the following two inequalities are used:

$$\left|\sum_k a_k\right| \leq \sum_k |a_k|$$

for a complex sequence a_k and

$$|A_K(f)\exp(j2\pi k f_R \tau)|\leq|A_K(f)|.$$

This can be summarized as

$$\sup_{\theta}\int_{-\infty}^{\infty}|H(\theta(\tau), f)-\text{avg}_{\theta}H(\theta, f)|^2 df \leq\int_{-\infty}^{\infty}\left[\sum_{k\neq 0}|A_k(f)|\right]^2 df \quad (19)$$

Similarly, the right-hand side of Eq. (18) can be expressed in terms of $A_k(f)$. Changing the order of two operators "avg" and "$\int$" gives

$$\text{avg}_{\theta}\int_{-\infty}^{\infty}|H(\theta, f)|^2 df=\int_{-\infty}^{\infty}\text{avg}_{\theta}|H(\theta, f)|^2 df$$
$$=\int_{-\infty}^{\infty}\text{avg}_{\tau}\left|\sum_{k=-\infty}^{\infty}A_k(f)\exp(j2\pi k f_R \tau)\right|^2 df$$
$$=\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}|A_k(f)|^2 df \quad (20)$$

where the avg" operator sifts only DC terms to arrive at the last equality. With Eq. (19) and Eq. (20), Eq. (18) can be transformed into the following:

$$\int_{-\infty}^{\infty}\left[\sum_{k\neq 0}|A_k(f)|\right]^2 df<\alpha\int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}|A_k(f)|^2 df \quad (21)$$

It is worth noting that Eq. (18) and Eq. (21) are not equivalent (see Eq. (19)). It is possible that Eq. (18) ("tight bound") may be satisfied, while Eq. (21) ("loose bound") is not. However, it is much simpler to evaluate Eq. (21) than Eq. (18). We propose to adjust the critical number α to achieve an optimal tradeoff between the tightness (of the bound) and simplicity. Earlier in this section when the sideband criterion was presented, the critical number α was a number given a priori. From Eq. (17), Eq. (18) and Eq. (21), we can conclude that α can be interpreted as a quotient between the variation energy and average energy. With $\alpha=0.2$, the LCS becomes a PLCS if the variation energy is less than 20% of the average energy. In practice, α should be chosen slightly larger than the threshold one wants to adopt, in order for Eq. (21) to be evaluated instead of unwieldy Eq. (18). There exist no general rules that specify the optimal α and the associated risk factor to take into account the looseness of Eq. (21) but our experience shows that the following heuristic rule works effectively:

1. Select α based on a priori knowledge.
2. Multiply α by the risk factor 1.5.

There is one remaining issue in evaluating Eq. (21): Eq. (21) is the continuous integral in the frequency domain for the freqeacy range of 0 to ∞. Although nontrivial theoretically, this problem can be relatively easily resolved experimentally. The goal is to approximate the integrals in Eq. (21) as arithmetic sums. The following experimental procedure is used:

1. Perform a swept-sine test with a spectrum analyzer at a sufficient frequency resolution so that any significan t change of the frequency response function is not undetected. The frequency span of the swept-sine test is determined by the frequency range of interest, i. e. active noise/vibration frequency band.

2. If the frequency response function does not change significantly over adjacent frequencies, decrease the frequency resolution. Otherwise,

maintain the frequency resolution.

3. At each frequency f of the swept-sine test, measure $A_k(f)$ as in Sec. 3.

4. With measured $A_k(f)$, evaluate Eq. (21).

The sideband criterion in this paper contains some nice features, among which it stands out that the criterion is evaluated in the frequency domain rather than in the time domain. Owing harmonic disturbances from the running machine, sensor/actuator noise, etc., it is very difficult to obtain disturbance-free data in the time domain. The frequency domain experiment can avoid the harmonic disturbances by probing at the frequencies between the harmonics. In addition, experiments in the frequency domain provide certain advantages over those in the time domain. First of all, the effective signal-to-noise ratio within the frequency range of interest can be significantly improved since the excitation energy can be concentrated within a narrow frequency band while the sensor/actuator noise tends to occupy a wide frequency band. Secondly, the higher signal-to-noise ratio for a given input excitation energy makes it possible to prevent any possible nonlinearity by lowering the level of the input signal, since the nonlinearity may distort the analysis (remember that the linear cyclic systems are considered in this paper).

6. Concluding Remarks

A generic expression for analyzing the LCS's is derived. The cyclic nature of the LCS is shown to generate a series of amplitude-modulated signals whose carrier frequencies are harmonic multiples of the fundamental frequency of the LCS. A criterion is developed to classify a given LCS into a GLCS or PLCS. The criterion provides a simple experimental test, from which the LCS can be classified. The criterion can be checked before signal processing techniques for the LTIS's are applied to analyze the rotating machinery, in order to validate the assumption that the dynamic system under consideration is LTI.

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